

[Invited Paper]

THE DEFINITION OF A GEODETIC DATUM

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Abstract: Some theoretical problems related to the concept of a geodetic datum are discussed: the definition of a global terrestrial rectangular coordinate system, the position of a datum with respect to the geocenter and its orientation with respect to the global axes, the role of the Laplace equation, and the choice of a reference ellipsoid.

Introduction

A geodetic datum is usually defined in terms of five parameters a , f ; x_0 , y_0 , z_0 . Here a and f denote semimajor axis and flattening of the reference ellipsoid, which is taken as an ellipsoid of revolution, and x_0 , y_0 , z_0 are the rectangular coordinates of the center of the reference ellipsoid with respect to the geocenter, the Earth's center of mass. This definition presupposes an underlying basic system of rectangular coordinates XYZ , the Z -axis coinciding with the mean rotation axis of the Earth, and the X -axis passing through the zero meridian, which is the mean Greenwich meridian. The rotation axis of the ellipsoid is supposed to be parallel to the Z -axis.

The geodetic coordinates ϕ (geodetic latitude), λ (geodetic longitude), and h (height above the reference ellipsoid) are then related to the rectangular coordinates XYZ by the well-known equations (cf. Heiskanen and Moritz 1967: 205)

$$\begin{aligned} X &= x_0 + (\nu + h) \cos \phi \cos \lambda, \\ Y &= y_0 + (\nu + h) \cos \phi \sin \lambda, \\ Z &= z_0 + ((b^2\nu/a^2) + h) \sin \phi, \end{aligned} \tag{1}$$

where

$$\nu = \frac{a^2}{\sqrt{a^2 \cos^2 \phi + b^2 \sin^2 \phi}} \tag{2}$$

is the east-west radius of curvature of the reference ellipsoid and b is its semiminor axis.

Contrary to the geodetic coordinates ϕ , λ , h , the natural coordinates Φ (astronomical latitude), Λ (astronomical longitude), and C (geopotential number) are directly measurable. The angles Φ and Λ define the direction of the plumbline of the observation station and can be determined by astronomical measurements. The astronomical longitude Λ is measured from the same zero meridian plane as geodetic longitude λ , so that, on the unit sphere, the meridians $\Lambda = 0$ and $\lambda = 0$ coincide by definition. The geopotential number C represents height above sea level; it may be replaced by the geometrically more significant, though less directly measurable, orthometric height H , which is the elevation above the geoid.

There is no direct simple mathematical relation, comparable to (1), between Φ , Λ , H and X , Y , Z because the natural coordinates are subject to, usually unknown, irregularities of the gravitational field. However, the differences between Φ , Λ , H and ϕ , λ , h are small; we put

$$\begin{aligned}\Phi &= \phi + \xi, \\ \Lambda &= \lambda + \eta \sec \phi, \\ H &= h - N;\end{aligned}\tag{3}$$

ξ and η are the components of the deflections of the vertical and N is the height of the geoid above the ellipsoid.

The present paper discusses theoretical aspects and problems connected with these concepts, especially with the definition of rectangular axes XYZ and of the reference ellipsoid underlying the geodetic coordinates ϕ , λ , h .

The Basic Rectangular Coordinate System

If the Earth were an ideally rigid body, then the definition of a basic system of rectangular coordinates would be relatively easy. Difficulties are introduced by the fact that the Earth deforms under the action of tides and that there are shifts of masses due to atmospheric motions, etc. Plate motion and similar effects further complicate the picture.

According to Munk and Macdonald (1960: 10–12), there are three principal possibilities for defining terrestrial rectangular axes, the first two of which are based on the laws of mechanics.

(1) Tisserand's mean axes of body are defined such that the relative angular momentum is zero in this system.

(2) The principal axes, or axes of figure form that rectangular system in which the tensor of inertia becomes a diagonal matrix.

(3) The "geographic" axes are attached in a prescribed way to a set of principal observatories, such as the stations of the International Polar Motion Service.

The first two definitions have the advantage of admitting a physical interpretation; the equation of motion for the Earth's masses, the Liouville equation, assumes particularly simple forms in these two systems.

For the present purpose they have the essential drawback that the coordinates of the observation stations are not fixed in these two systems, not even in an average sense, after removing tidal motion. Hence it is better to use "geographic" axes, in which the observatories are at rest on the average (it is appropriate to remove beforehand the short-periodic effects of the lunar-solar tide as far as possible, using a suitable tidal model).

The basic source of information regarding a precise definition of such terrestrial reference systems and their realization are the Proceedings of the Colloquium on Reference Coordinate Systems for Earth Dynamics held in Toruń, Poland in August 1974 (Kolaczek and Weiffenbach 1975), which contain many papers concerning this question.

The subject is particularly difficult and controversial if we aim at an accuracy level of 10^{-8} , which may become relevant in the future. To an accuracy of 10^{-6} , corresponding to a precise triangulation, the standard definitions (Mueller 1969: 337 and 351) seem to be fully sufficient: the Z-axis has the direction of the CIO (Conventional International Origin, defining a mean pole), and the X-axis is parallel to the Zero Meridian adopted by BIH (Bureau International de l'Heure), also denoted as the "Greenwich Mean Astronomic Meridian" (it is conventionally defined and has no direct relation to the Greenwich observatory); these axes are a practical realization of "geographical axes" in the above-mentioned sense.

Plate motion and the effect of atmospheric and ocean circulation are too small to have an influence in the present context (apart from exceptional effects along the San Andreas Fault, etc.). The same holds for tidal effects. It is true that tidal displacements of points on the Earth's surface may reach several decimeters, but neighboring points move in much the same way, so that the effect on measured distances and angles is far below the measuring accuracy. Only on precise leveling may there be noticeable effects which, however, can be readily corrected for.

Datum Shift and Orientation

Datum Shift. The quantities x_0 , y_0 , z_0 in (1) represent the coordinates of the center of the ellipsoid with respect to the geocenter; they are called "shift parameters." They are in principle inaccessible to determination by geometric techniques—astrogeodetic method, three-dimensional terrestrial triangulation, satellite triangulation—but they can be determined by physical methods: the gravimetric method and dynamical satellite techniques. Using one of the latter methods it is thus possible to place the center of the reference ellipsoid at the geocenter, which is certainly the natural position, thus obtaining a geocentric datum.

From a practical point of view, dynamical satellite methods are definitely superior; an accuracy of a few meters or better (there is some doubt about possible systematic effects) has been achieved in the shift components. The gravimetric method, using Stokes' and Vening Meinesz integral formulas, suffers from the lack of uniform global gravity coverage. The accuracy in a vertical direction, determined by the accuracy of N , may be of a comparable order of magnitude, a few meters standard error, but the accuracy of shift determination in a horizontal direction, defined by the accuracy of absolute (i.e., geocentric) deflections of the vertical is definitely inferior: ± 3 m accuracy in horizontal position would

correspond to $\pm 0.1''$ in ξ and η , of which presently obtainable accuracies fall short by a factor of at least 5.

A geocentric positioning of a datum to an accuracy of a couple of meters or better, achievable by satellite methods, especially Doppler, corresponds to a relative accuracy better than 10^{-6} , which is in keeping with accuracy goals in terrestrial networks. Therefore, a geocentric positioning of the reference ellipsoid can be made and should be made by all means in the best possible way.

It is the opinion of the author that nongeocentric datums are a thing of the past and that modern datums should be geocentric. Apparent advantages of nongeocentric datums, especially possibly slightly better local fit of the geoid by a nongeocentric reference ellipsoid, certainly do not outweigh the advantages of a geocentric system: theoretically, a physically meaningful and unambiguous definition of the coordinate origin, and practically, an immediate relation to global reference systems such as provided by Doppler.

Orientation. Let us write equations (1) in the form

$$\begin{aligned} X &= x_0 + x, \\ Y &= y_0 + y, \\ Z &= z_0 + z, \end{aligned} \tag{4}$$

where

$$\begin{aligned} x &= (\nu + h) \cos \phi \cos \lambda, \\ y &= (\nu + h) \cos \phi \sin \lambda, \\ z &= ((b^2\nu/a^2) + h) \sin \phi. \end{aligned} \tag{5}$$

Then X, Y, Z are geocentric rectangular coordinates, and x, y, z are ellipsoidal rectangular coordinates, the origin being at the center of the reference ellipsoid.

In the highly desirable case of a geocentric datum just considered, there holds $x_0 = y_0 = z_0 = 0$ and $x = X, y = Y, z = Z$, but for the sake of generality we shall admit also nonzero x_0, y_0, z_0 in the following discussion.

The "geocentric" axes XYZ and the "ellipsoidal" axes xyz are parallel: the two systems differ only (possibly) by a shift, but not by a rotation.

In an obvious vector notation we may abbreviate (4) as

$$\underline{X} = \underline{x}_0 + \underline{x}. \tag{6}$$

In the presence of a rotation of the two systems XYZ and xyz with respect to one another, (6) would have to be replaced by

$$\underline{X} = \underline{x}_0 + \underline{R}\underline{x}, \tag{7}$$

where \underline{R} is a rotation matrix. If the rotation is only small, we may write

$$\underline{R} = \underline{I} + \delta\underline{R}, \tag{8}$$

\underline{I} being the unit matrix and $\delta\underline{R}$ is an “infinitesimal” rotation matrix of form

$$\delta R = \begin{bmatrix} 0 & \epsilon_3 & -\epsilon_2 \\ -\epsilon_3 & 0 & \epsilon_1 \\ \epsilon_2 & -\epsilon_1 & 0 \end{bmatrix} . \quad (9)$$

Again, for similar theoretical and practical reasons as for geocentricity, the axes xyz and XYZ should be parallel as accurately as possible ($\underline{R} = \underline{I}$); if, in addition, $x_0 = y_0 = z_0 = 0$, then the two systems will coincide.

In contrast to geocentricity, the parallelism of coordinate axes can be realized by geometric means. The so-called Laplace equation has long been known to achieve this already for classical triangulation. The basic significance of this equation merits a separate discussion, which will be given in the following section.

Because of random and existing systematic errors, especially older, geodetic datums may not be free from rotation, as well as contain a datum shift. We may then, from ϕ, λ, h referred to this datum, compute x, y, z by (5) and compare them to geocentric coordinates X, Y, Z in the basic rectangular system, as obtained, e.g., by Doppler methods. If we write (7) for (at least) two points, we get (at least) six equations, which may be solved for the shift parameters x_0, y_0, z_0 and the rotation parameters $\epsilon_1, \epsilon_2, \epsilon_3$.

To repeat, however, for a modern geodetic datum all these six parameters should be made zero as accurately as possible, e.g., to better than 10^{-6} .

The Role of the Laplace Equation

The lengthy elementary discussion in this section is intended to clarify some points recently raised; the reader not interested in these technical details is advised to pass on to section 5.

Assume that at a point P on the Earth’s surface, the astronomical azimuth A and the zenith distance ζ' to another point Q has been measured. Both A and ζ' refer to the actual plumbline at P , since the vertical axis of the theodolite is made to coincide with this plumbline. If, instead, the theodolite axis could be made to coincide with the normal, at P , to the reference ellipsoid, then we would measure a “geodetic” azimuth α and a “geodetic” zenith distance ζ . Astronomical azimuth A and astronomical zenith distance ζ' are related to their geodetic counterparts by

$$A - \alpha = \eta \tan \phi + (\xi \sin \alpha - \eta \cos \alpha) \cot \zeta, \quad (10)$$

$$\zeta' - \zeta = -(\xi \cos \alpha + \eta \sin \alpha). \quad (11)$$

An elementary derivation is bound in Heiskanen and Moritz (1967: 186 and 190); an elegant matrix derivation is given in Hotine (1969: 134).

Especially in first-order triangulation, all lines of sight will usually be almost horizontal, that is, $\zeta = 90^\circ$. Then (10) reduces to

$$A - \alpha = \eta \tan \phi. \quad (12)$$

On using the second equation of (3), this becomes

$$A - \alpha = (\Lambda - \lambda) \sin \phi \quad (13)$$

or, briefly,

$$\Delta \alpha = \Delta \lambda \sin \phi . \quad (14)$$

Equation (13) or (14) is called Laplace's equation, or Laplace condition. If it is possible, at a station, to observe astronomically both longitude Λ and azimuth A , and if their geodetic counterparts λ and α can be found independently (by the computation of the triangulation), then (13) forms a condition which the four quantities A , α , Λ , λ must satisfy.

It is clear that, if ζ cannot be put 90° , then the exact form of the Laplace equation, by (3) and (10), is

$$A - \alpha = (\Lambda - \lambda) \sin \phi + [(\Phi - \phi) \sin \alpha - (\Lambda - \lambda) \cos \phi \cos \alpha] \cot \zeta. \quad (15)$$

All these equations have been derived on the basis of parallelism of the ellipsoidal axes xyz and the global axes XYZ . Their use in the computation of a triangulation will, therefore, serve to ensure the parallelism of these two rectangular frames.

To understand the situation, let us briefly consider, from a geometric point of view, how a triangulation is computed. To make the geometric structure transparent, we presuppose errorless measurements.

Let us first assume, for the sake of simplicity, that all stations of the triangulation lie on the surface of the reference ellipsoid (as a second step, we shall rid ourselves of this oversimplification). However, the plumb lines are not taken to coincide with the ellipsoidal normal; that is, deflections of the vertical ξ , η are admitted. Distances (straight spatial distances, i.e., chords between ellipsoidal points) and horizontal angles are given to the extent necessary to determine the geometric configuration on the ellipsoid. Astronomical coordinates Φ and Λ have been observed at each station. At one station, the initial point P_0 , an astronomical azimuth A_{01} has been measured in addition to Φ_0 and Λ_0 . For P_0 , also geocentric coordinates X_0, Y_0, Z_0 are given.

The geodetic coordinates ϕ and λ of the triangulation points (h is zero by our assumption) can be obtained as follows. Let us abbreviate (1), with $\underline{x}_0 = 0$ (geocentric datum), in the form

$$\underline{X} = \underline{X}(\phi, \lambda, h).$$

For the initial point P_0 this becomes

$$\underline{X}_0 = \underline{X}(\phi_0, \lambda_0, h_0).$$

These equations are solved for ϕ_0, λ_0, h_0 ; since the initial point P_0 is on the ellipsoid, there must be $h_0 = 0$ with our errorless data.

With the measured astronomical coordinates Φ_0 and Λ_0 , we can compute deflections of the vertical at P_0 by (3):

$$\xi_0 = \Phi_0 - \phi_0, \eta_0 = (\Lambda_0 - \lambda_0)\cos \phi_0.$$

Then (10), which practically reduces to (12), gives

$$\alpha_{01} = A_{01} - \eta_0 \tan \phi_0, \tag{16}$$

determining the initial ellipsoidal azimuth α_{01} from the measured astronomical azimuth A_{01} .

Horizontal angles measured at P_0 can also be reduced to the ellipsoid:

$$\begin{aligned} \gamma_{12}^{el} = \gamma_{12}^{meas} &+ (\xi_0 \sin \alpha_{01} - \eta_0 \cos \alpha_{01}) \cot \zeta_{01} \\ &- (\xi_0 \sin \alpha_{02} - \eta_0 \cos \alpha_{02}) \cot \zeta_{02}, \end{aligned} \tag{17}$$

by taking the difference of two equations (10). The main term $\eta \tan \phi$ has dropped out, so that for nearly horizontal lines of sight the reduction of horizontal angles will be very small and often negligible.

Now the azimuths of all directions $P_0 P_1, P_0 P_2, \dots$ initiating from P_0 can be computed (fig. 1):

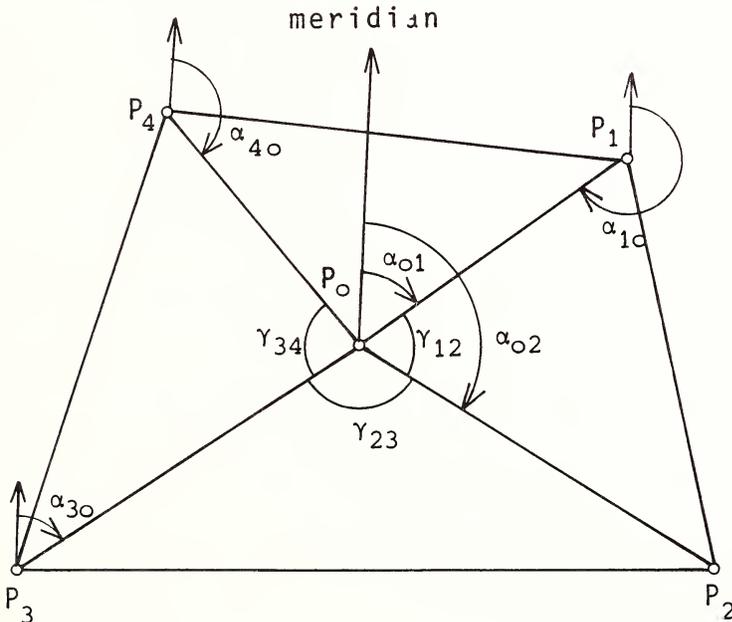


FIGURE 1.

α_{01} by (16)

$$\alpha_{02} = \alpha_{01} + \gamma_{12}$$

$$\alpha_{03} = \alpha_{02} + \gamma_{23}$$

(18)

where γ_{ij} denote ellipsoidal values γ_{ij}^{ell} .

By well-known ellipsoidal computation methods using chords (Molodenskii et al. 1962: chapter I, § 4) we can then compute the geodetic coordinates of the triangulation points surrounding P_0 (in the figure: P_1, P_2, P_3, P_4) and finally the reverse azimuths $\alpha_{10}, \alpha_{20}, \alpha_{30}, \alpha_{40}$ (without any additional azimuth measurements!).

The next point, say P_1 , can be treated in exactly the same way. The previous ellipsoidal computations have given $\phi_1, \lambda_1, \alpha_{10}$; from astronomical observations we know Φ_1, Λ_1 , so that (3) gives ξ_1, η_1 . Then the horizontal angles measured in P_1 can be reduced to the ellipsoid (if necessary) and we can proceed to the points surrounding P_1 , and so on.

In this way we obtain geodetic coordinates ϕ, λ of all stations and geodetic azimuths α of all sides.

The essential point is that only one astronomically measured azimuth (in our case A_{01}) is needed to completely fix the orientation of the triangulation, the parallelism of the ellipsoidal and terrestrial Z -axis is ensured by the choice of the geodetic datum.

What happens if another astronomical azimuth, say A_{56} , is measured? Since $\Phi_5, \Lambda_5, \phi_5, \lambda_5$ are known according to the foregoing considerations, we get ξ_5 and η_5 and can thus compute α_{56} by (10). However, we already know α_{56} from the preceding ellipsoidal computations. Thus the two values for α_{56} , one from ellipsoidal computations and one from reduction of the measured A_{56} to the ellipsoid, must coincide. In other terms, the Laplace condition (14) must be satisfied

$$(A_{56} - \alpha_{56}) = (\Lambda_5 - \lambda_5) \sin \phi_5, \quad (19)$$

α_{56} being the value from ellipsoidal computations (more exact is, of course, (15)).

If our observations are errorless, as we have assumed, then this Laplace equation will automatically be satisfied because the geometric situation is uniquely determined.

If the observations are affected by measuring errors, then the Laplace condition will no longer be satisfied, but can be enforced by an adjustment. Any additional astronomical azimuth measurement gives another Laplace condition for adjustment. In this way the ideal geometrical condition (exact orientation and parallelism of coordinate axes) can be restored to the best possible extent.

It is in this sense that the assertion "the Laplace condition ensures net orientation and parallelism of axes" is to be understood: it is a condition, not for fixing the geometry, but for adjusting measuring errors.

So far we have assumed that the triangulation points lie on the surface of the ellipsoid. We shall now free ourselves from this assumption: the points are now situated on the Earth's surface. Therefore, we need additional information on their heights. Two principal methods are available: (1) the astrogeodetic method and (2) three dimensional triangulation using zenith distances.

The second method is theoretically more attractive, but suffers from practical difficulties with zenith distances. So the first method is preferred in practice.

In the astrogeodetic method, geoid height determination by astronomic levelling, combined with orthometric heights, is used to reduce this case to the first simple case of points on the ellipsoid by an iterative procedure (Heiskanen

and Moritz 1967: 197–199). The geometrical situation with respect to the Laplace equation remains essentially the same.

In three-dimensional triangulation, we may reduce the observations (azimuths, horizontal angles, zenith distances) to the ellipsoidal normal and use geodetic coordinates (*ibid.*: 223–224). In this way, the situation is again brought back to the first case, the Helmert projections of the triangulation points onto the ellipsoid playing the role of the former points situated on the ellipsoid. Again the geometric situation with respect to the Laplace azimuth equation remains the same. One astronomical azimuth is needed to fix the geometric orientation, other azimuths help to adjust for measuring errors and thus to produce a geometry that approaches as well as possible the ideal case: proper orientation and parallelism.

If, in this case, more zenith distances are measured as it is geometrically necessary, then the surplus measurements provide conditions of the form (11). In this sense it is understandable that the system of the two equations (10) and (11) has recently been called “extended Laplace equation.” However, such conditions on zenith distances only help to improve heights; they have practically no effect on orientation and axes. Therefore, the present author prefers to reserve the name “Laplace equation” to the azimuth condition.

It has sometimes been asked how many Laplace conditions are necessary to determine the geometric orientation (Hotine 1969: 134). In the author’s opinion, this question should not be put in this way, because it is ambiguous: the answer may run from zero (if only surplus conditions are considered) to any number (if the reduction of any horizontal or vertical angle measurement is viewed as an application of the “extended Laplace equation” (10) and (11)).

It seems clearer to ask for the number of measurements necessary to fix the geometry: Φ and Λ at each point, enough horizontal angles, distances and (as the case may be) zenith distances to determine the geometrical configuration, and one astronomical azimuth to fix orientation. A total number of n astronomical azimuths (at n different points) gives $n - 1$ Laplace conditions, geometrically superfluous but practically of the greatest importance. The more and the better distributed these azimuths are, the better will be the orientation of the network and of the coordinate axes.

The Reference Ellipsoid

Contemporary geodetic reference systems use an ellipsoid of revolution as a reference surface. Why? Why do we need an ellipsoid at all, and why should we not use a surface defined by a higher degree spherical harmonic expansion?

A reference system is, by its very meaning, a simplified system to which to refer the real situation if reality is too complicated for a simple direct description. A reference system should thus be simple. On the other hand, it should represent an idealized, simplified version of reality. It should thus be reasonably realistic; in this way it may even be used to replace the realistic situation if the accuracy requirements are low. It should be comprehensive in the sense that it covers as many different relevant phenomena as possible. Still, these requirements do not determine uniquely our reference system; a certain amount of arbitrariness remains, the system will, to a certain extent, be conventional. Still, it should be well defined.

These features are well illustrated by the case of the reference ellipsoid. A suitable ellipsoid of revolution is the simplest geometrical surface that approximates well the geoid (to 10^{-5} ; maximum deviations are on the order of 100 m). If the ellipsoid is equipped with a normal gravity field by postulating it to be an equipotential surface, then this "level ellipsoid" serves as a reference as well for the geometry of the Earth's surface as for the terrestrial gravity field; it thus provides a comprehensive geodetic reference system. To make the ellipsoidal system well defined, its center should be placed at the geocenter and its axes should be oriented to agree with the basic terrestrial axes *XYZ* discussed in section 1. The best fit of the Earth by an ellipsoid may be formulated in different ways, by various minimum principles or by identifying four ellipsoidal parameters with the corresponding terrestrial ones (Heiskanen and Moritz 1967: 216–217); this introduces a certain amount of conventionality, which is heightened by the fact that a reference ellipsoid need not even be a best fitting ellipsoid. (See below.) On the other hand, it is gratifying that many different definitions of best fit give practically the same results.

Simplicity is not only a pleasant esthetic feature. It helps the user, who can understand the ellipsoid without having to bother with the intricacies of physical geodesy. (Simplicity also helps us theoreticians to understand a certain problem: how often are we misled by complicated details and reach understanding only by the right simplification.)

Evidently, simplicity is not absolute; otherwise we would use the basic global rectangular system *XYZ* without an ellipsoid. This has been sometimes suggested, but what about approximation of the Earth and what about the surveyor who is to use such a system?

Simplicity is important. This is why an ellipsoid of revolution will not be superseded, as a basic reference, by a three-axial ellipsoid or by a level surface defined by a truncated spherical-harmonic expansion. The advantage of these surfaces to be slightly more "realistic" is far outweighed by the complication thereby introduced.

There is also a closely related argument. A reference is as good as its being universally accepted and widely used. It is feasible to get general agreement on a simple system involving few parameters, such as an ellipsoid of revolution defined geometrically and physically by four parameters. Getting general agreement on a system defined by a complicated theory and by 25 numerical parameters is one of the nightmares haunting the sleep of the chairman of a study group on geodetic constants.

This does not, of course, preclude the additional use of more complicated systems as auxiliary references for special purposes, for instance, a spherical-harmonic expansion in least-squares collocation.

Small irregular effects can often be conveniently handled by applying corrections to the data. Thermal effects on measuring rods and atmospheric influences on electromagnetic distance measurements are naturally handled in practice in this way (rather than working with a geometry that is not Euclidean). On the same logical footing, though perhaps less obvious, is the consideration of the atmospheric influences on gravity measurements by suitable corrections, instead of burdening the reference ellipsoid with an atmosphere. Therefore, in the

Geodetic Reference System 1967 a reference ellipsoid without atmosphere was used (Levallois 1970), and also future references should use this principle.

Numerical Values. The Geodetic Reference System 1967 has the following four defining parameters:

$$\begin{aligned} a &= 6378\,160\,m, \\ GM &= 3.986\,03 \times 10^{14}\,m^3\,s^{-2}, \\ J_2 &= 0.001\,0827, \\ \omega &= 7.292\,115\,1467 \times 10^{-5}\,rad \cdot s^{-1}, \end{aligned}$$

where

$$\begin{aligned} a &= \text{semimajor axis,} \\ GM &= \text{geocentric gravitational constant} \\ &\quad (\text{Newtonian constant } G \text{ times mass } M \\ &\quad \text{of Earth including atmosphere),} \\ J_2 &= \text{zonal spherical-harmonic coefficient} \\ &\quad \text{of second degree,} \\ \omega &= \text{angular velocity of the Earth's rotation.} \end{aligned}$$

From these defining constants, other parameters can be unambiguously derived, for instance, the flattening

$$f = 1/298.247$$

and equatorial gravity

$$\gamma_e = 9.780\,318\,ms^{-2}$$

(rounded values).

The Department of Defense World Geodetic System 1972 (Seppelin 1974) has

$$\begin{aligned} a &= 6378\,135\,m, \\ GM &= 3.986\,008 \times 10^{14}\,m^3\,s^{-2}, \\ J_2 &= 0.001\,082\,616, \\ \omega &= \text{as above,} \\ f &= 1/298.26, \\ \gamma_e &= 9.780\,333\,ms^{-2} \end{aligned}$$

At the XVth General Assembly in Grenoble 1975, the IUGG recommended the following "currently representative estimates"

$$\begin{aligned} a &= (6378\,140 \pm 5)\,m, \\ GM &= (3986\,005 \pm 3) \times 10^8\,m^3\,s^{-2}, \\ J_2 &= (108\,263 \pm 1) \times 10^{-8}, \\ 1/f &= (298\,257 \pm 1.5) \times 10^{-3}, \\ \gamma_e &= (978\,032 \pm 1) \times 10^{-5}\,ms^{-2}, \end{aligned}$$

ω being again the same as in the Geodetic Reference System 1967 (Moritz 1975).

These last values have been adopted by the International Astronomical Union at its General Assembly in Grenoble 1976 as part of the new System of Astronomical Constants.

The Geodetic Reference System 1967 has not been officially changed in 1975. IAG plans to revise fundamental geodetic constants, including Earth ellipsoid parameters, every 4 years, adopting a list of such values, currently considered representative, at each General Assembly. On the other hand, a geodetic reference system by no means needs to incorporate the currently best values. In fact, if all is correctly done, the final results (spatial position of triangulation points, the geoid, etc.) will be independent of the particular choice of the ellipsoidal reference system (as long as it is within reasonable limits), which thus plays only an intermediate role without having any effect on the final result. So the desire to have a reference ellipsoid that fits the geoid as closely as possible is motivated primarily by esthetical reasons. Practically much more important is long-term stability; an enormous amount of data is based on an adopted reference system, and such a reference should be changed as infrequently as possible.

It seems, however, that the next IUGG General Assembly in 1979 may be the right time to change the Geodetic Reference System 1967. Already now, the basic parameters are known to an accuracy determining the ellipsoid to 10^{-6} or better. The values currently (1978) considered best differ very little from the 1975 values (but greatly from the 1967 parameters) IAU has adopted a new System of Astronomical Constants, and the new adjustment of the triangulation of a whole continent may give practical stability and long life to a new IAG reference system, provided it is used here and by other countries.

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Discussion

Ashkenazi: Do you advocate the adoption of a geocentric coordinate system for all geodetic work, i.e., even for small countries where the geocentric system may not be the most convenient from an approximation point of view?

Moritz: Yes.

Bossler: Would you care to enumerate the disadvantages of a local system?

Moritz: Essentially, it is a question of whether or not we should use a global or local datum. A global datum ("World Geodetic System") corresponds to taking semi-major axis a and flattening f from a globally best fitting ellipsoid and using zero shift components ($x_0 = y_0 = z_0 = 0$), that is, a geocentric datum.

By taking somewhat different values of a and f and nonzero shift components, one could obtain a slightly better local fit of some geoidal region by an ellipsoid. Why, then, use a global datum?

Let me try to make this clear by an analogy. For a local survey of a city, considered by itself, a local coordinate system xy has definite advantages: less distortion, a convenient choice of coordinate axes and origin, and no efforts needed to relate to a national triangulation. Still, a regional or national system is considered to be much better: uniformity, compatibility of local surveys and maps, no problem of transition between different systems, and the like. These larger advantages far outweigh the immediate advantages of local systems.

We have a similar situation in global geodesy. A uniform global reference system for all countries will permit immediate compatibility of coordinates, charts, and so on. A new adjustment of a continent must be seen in such a larger perspective.

If one wished to use a nongeocentric system to get a better fit, then it would be only logical also to employ locally best fitting values of a and f . Again, if the NAD were a local datum, then there is no reason for other countries not to use individual local systems which are best fitting for them. Thus, we would be back to the old patchwork of local systems, and a great chance for modern geodesy would be lost.

Chovitz: The 1967 system adopted by the International Association of Geodesy is almost certainly going to be changed in 1979 just as the International Astronomic Union did in 1976.